

## Lesson 4

## CAPACITANCE AND INDUCTANCE

**Task.** The skills and knowledge taught in this lesson are common to all missile repairer tasks.

**Objectives.** When you have completed this lesson, you should be able to explain what capacitance and inductance are and describe resistive and capacitive circuits and resistive and inductive circuits.

**Conditions.** You will have this subcourse book and work without supervision.

**Standard.** You must score at least 70 on the end-of-course examination that covers this lesson and lessons 1, 2, and 3 (answer 27 of the 38 questions correctly).

**CAPACITANCE**

In mechanical systems, if an elastic body such as a rubber band is stretched, there is little or no opposition during the first moment of movement. The opposition increases as the rubber band is extended. If the rubber band is stretched and then fixed in position, the work done in stretching it is stored in the band and is returned when the rubber band is released. This mechanical property of elastic bodies illustrates the electrical property called capacitance.

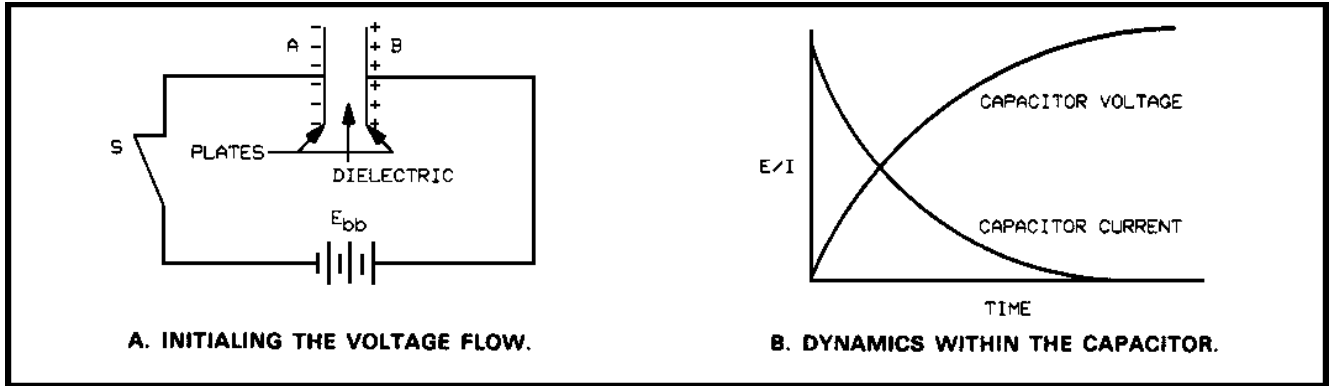
A capacitor is any two conductors separated by an insulating material. When a DC voltage is first applied across a capacitor, considering no losses, the current flow is maximum the first instant and then rapidly decreases to zero when the voltage developed across the capacitor equals the applied voltage. Energy was consumed in building up the voltage. Removal of the applied voltage leaves the capacitor in charged condition, and energy is stored in the capacitor.

The conductors mentioned above are called the plates of the capacitor, and the insulating material is called the dielectric. In figure 4-1A, when the switch(s) is closed, the battery voltage will cause current to flow. The current flow will be maximum the first instant and will decrease as the voltage across the capacitor builds up. This is shown in figure 4-1B. The number of electrons flowing to plate A is equal to the number flowing away from plate B. The capacitor is fully charged when the difference of potential existing across the plates of the capacitor equals the battery voltage. At this time, the current through the capacitor is zero.

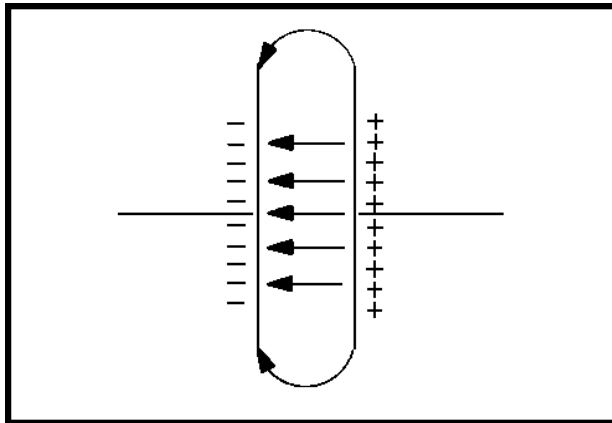
After the capacitor is fully charged, the switch may be opened and the capacitor will remain charged since there is no path for the electrons on plate A to reach plate B. The electrostatic field exists between the plates because they are two charged bodies. The electrical energy is stored between the plates. The direction of an electrostatic field has been arbitrarily stated as the direction a unit positive charge would move if placed in that field. The electrostatic field of the charged capacitor of figure 4-1 is shown in figure 4-2.

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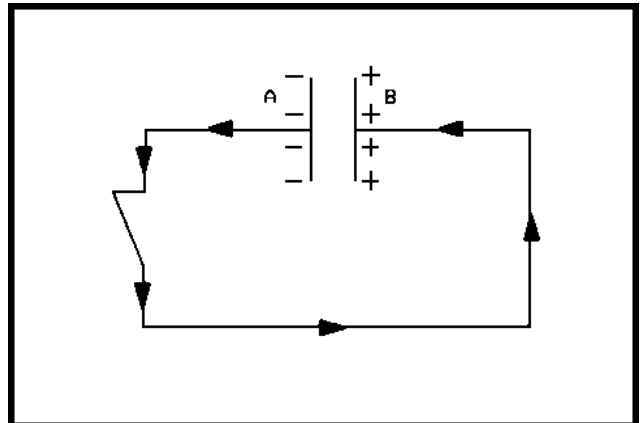
When a conductor is placed across the plates, the electrons flow from A to B (figure 4-3) returning the capacitor to its uncharged condition. This discharges the capacitor.



*Figure 4-1. Charging of a Capacitor.*



*Figure 4-2. Electrostatic Field Within a Capacitor.*



*Figure 4-3. Discharging of a Capacitor.*

**Measure of Capacitance**

The ability of a capacitor (C) to store electrical energy is called capacitance. The unit of capacitance is the farad (f), named for Michael Faraday. One f is the amount of capacitance that will store 1 coulomb of charge ( $6.28 \times 10^{18}$  electrons) when 1 V is applied. The farad is a very large unit, so the microfarad ( $\mu\text{fd}$ ) and picofarad (pfd) are commonly used. One microfarad equals  $10^{-6}$  f, and one pfd equals  $10^{-12}$  fd. Capacitors in common use range from 1 pfd to several thousand  $\mu\text{fd}$ .

The relationship between charge (Q), voltage (E), and C may be expressed as:

$Q = CE,$   
 where  
 Q = charge in coulombs,  
 C = capacitance in fd, and  
 E = voltage in V.

This relationship shows that an increase in charge (coulombs) will result from an increase in voltage or capacitance.

**Physical Characteristics**

**Relative Size of Parts.** The capacitance of a capacitor is determined by its construction. Capacitance is directly proportional to the area of its plates and inversely proportional to the thickness of the dielectric. This means that, if the plates were made larger, the capacitance would increase. If the plates were moved further apart, the capacitance of the capacitor would decrease.

**Dielectric.** The dielectric also affects capacitance because the energy is stored in the dielectric. For example, if a certain capacitor using air as the dielectric were immersed in oil, the capacitance would increase to three times as much. We say that oil has a dielectric constant of 3. Air has a dielectric constant of 1 and is the standard of comparison. The relationship of capacitance, area, thickness of dielectric, and dielectric constant is as follows:

$$C = \frac{Ak}{t}$$

- A = area of plate
- k = dielectric constant
- t = thickness of dielectric

The actual capacitance in pfd is

$$C = 0.2248 \frac{Ak}{t},$$

where A is in square inches and t is inches. The constant, 0.2248, in the above equation is necessary to convert from the electrostatic system of measurement (statcoulombs, statvolts, centimeters, etc.) to the more practical system, using such terms as ohms, amperes, coulombs, and inches.

The k of some common insulating materials are given below:

<i>Material</i>	<i>k</i>	<i>Material</i>	<i>k</i>
<b>Air</b>	1.	<b>Lucite</b>	2.5 to 3.
<b>Mica</b>	2.5 to 8.	<b>Quartz</b>	4. to 5.
<b>Bakelite</b>	5. to 9.	<b>Paper</b>	2. to 3.
<b>Porcelain</b>	7.		

The choice and thickness of the dielectric material determines not only the capacitance but also the voltage rating of the capacitor. Since it is usually desirable to have relatively large capacitance in a small space, the dielectric is quite thin. It must have high dielectric strength to avoid puncture by the applied voltage. Capacitors are commonly rated as to capacitance, working voltage, and peak voltage. DC working voltage is the maximum DC voltage the capacitor will stand satisfactorily under certain applied conditions (in its normal temperature range). AC working voltage rating is lower than DC working voltage rating for a given capacitor because the voltage used is the effective line voltage. For example, a capacitor with an AC working voltage rating of

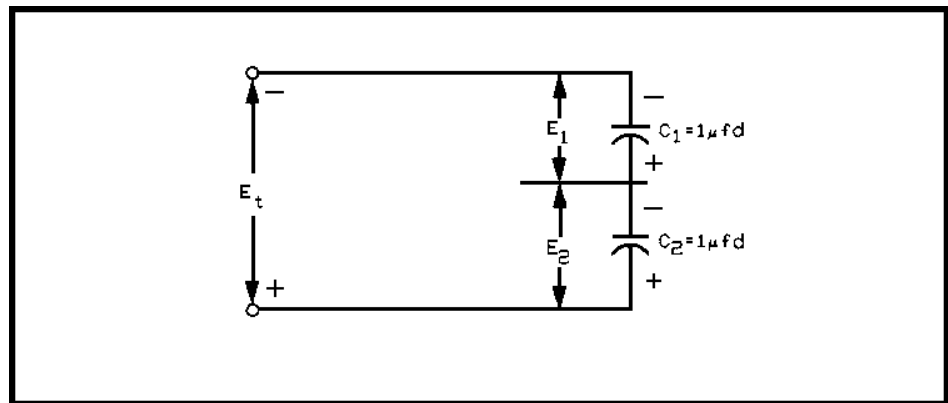
400 V must be able to withstand a peak voltage of 566 V because that is the peak voltage of the 400 VAC. Peak voltage ratings are generally higher than working voltage since most dielectrics can withstand higher voltages for a short time than they can stand continuously.

### Series Connection

You saw earlier that

$$C = 0.2248 \frac{Ak}{t},$$

where  $t$  represents the thickness of the dielectric. If the value of  $t$  is doubled, the capacitance would be reduced to one half. Connecting two equal capacitors in series is the equivalent of building one capacitor having a doubled thickness of dielectric. See figure 4-4.



**Figure 4-4. Capacitors in Series.**

When the upper plate of  $C_1$  is made negative, the lower plate of  $C_1$  must become equally positive. The same holds true for  $C_2$ . Since the upper plate of  $C_2$  is connected to the lower plate of  $C_1$ , these two points are electrically the same and effectively bring the two dielectrics together forming a single capacitor with a doubled thickness of dielectric. In this series circuit, as in all series circuits, the current flow is the same in all parts of the circuit. As a result, the same amount of charge will accumulate on each capacitor, thus the voltage is equally divided between the two capacitors.

Calculation of total series capacitance becomes more difficult when capacitors of unequal value are connected in series. In figure 4-4, two capacitors of equal value are connected in series. By Kirchhoff's law, the sum of the individual voltages must equal the applied voltage. The current is the same in all parts of the circuit. Since the current is the same, the charge deposited on each capacitor will be the same ( $Q=It$ ). Charge, voltage, and capacitance are related by

$$Q = CE \text{ or } E = \frac{Q}{C}.$$

This shows that the voltage on a capacitor is directly proportional to the charge and inversely proportional to the capacitance. The smallest capacitor will have the greatest voltage across it.

Substituting the value  $Q/C$  in place of  $E$ , in Kirchhoff's law for series circuits ( $E_t = E_1 + E_2$ ) you get

$$\frac{Q}{C_t} = \frac{Q}{C_1} + \frac{Q}{C_2},$$

Dividing both sides of this equation by  $Q$ , you get

$$\frac{1}{C_t} = \frac{1}{C_1} + \frac{1}{C_2}.$$

You can use this equation for determining the total series capacitance of any number of capacitors in series.

$$\frac{1}{C_t} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n}$$

You can rearrange equation as follows:

$$\begin{aligned} \frac{1}{C_t} &= \frac{1}{C_1} + \frac{1}{C_2} \\ \frac{1}{C_t} &= \frac{C_2 + C_1}{C_1 C_2} \\ C_t &= \frac{C_1 C_2}{C_1 + C_2} \end{aligned}$$

This equation is a useful and timesaving device, but may be used only for two capacitors in series.

Capacitors are often connected in series to allow their being used in circuits having a higher voltage than that for which they are rated. For instance, four 500-V, 4- $\mu$ fd units in series are the equivalent of a single 2,000-V, 1- $\mu$ fd unit. Series operation of capacitors of different values should be avoided. Since all units receive the same amount of charge, the voltage is divided in inverse proportion to the capacitance of each unit. One thousand V applied to 1  $\mu$ fd and 4  $\mu$ fd in series will produce 200 V across the larger capacitor and 800 V across the smaller. If both were rated at 500 V, the 1- $\mu$ fd capacitor would promptly break down putting the entire 1,000 V on the 4- $\mu$ fd capacitor, which would also be punctured.

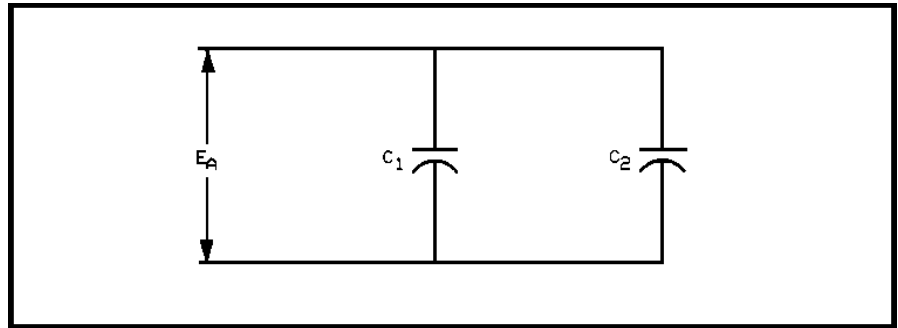
### Parallel Connection

Two equal capacitors in parallel are the equivalent of a single capacitor having doubled plate area and the same dielectric thickness (figure 4-5). The total capacitance, therefore, is the sum of the parallel capacitance, while the voltage applied must be no higher than the rating of the individual units. For any number of parallel capacitors, total capacitance is

$$C_T = C_1 + C_2 + C_3 + C_4 + \dots + C_n.$$

### Capacitive Reactance

Since capacitors are used in AC circuits, it is necessary to define the farad in terms of a changing voltage. When a steady voltage is applied to a capacitor, a momentary charging current flows. This current ceases almost immediately.



**Figure 4-5. Capacitors in Parallel.**

If the voltage were increased or decreased, another momentary flow of current would take place. If the voltage could be constantly changed, a continuous current would flow. A more rapidly changing voltage would cause a larger average current. For a given rate of voltage change, a larger capacitance causes a larger current to flow. The unit of capacitance (fd) may then be defined in terms of a changing voltage.

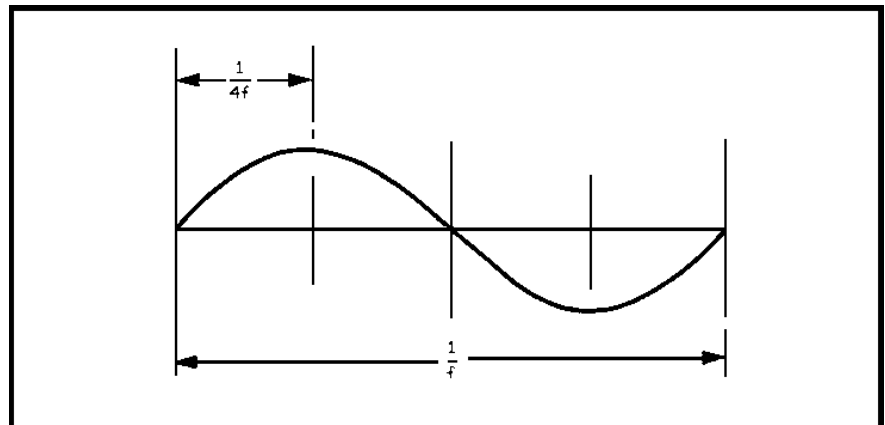
A capacitor is said to have a capacitance of 1 fd when a change of 1 V per second produces a current of 1 amp. This can be expressed as the equation,

$$I_{av} = C \frac{E}{t}$$

where  $I_{av}$  is the average current that is forced to flow through a capacitance of  $C$  fd by a change of  $E$  V in  $t$  sec.

In figure 4-6, a sine wave has been divided into 4 equal parts along the time base. The time required for one cycle is  $1/f$ . The time required for a change of voltage, from zero to  $E_{max}$ , is a quarter cycle, or  $1/4f$ . If a sine wave of voltage with a peak voltage of  $E_{max}$  and a frequency  $f$  is applied across a capacitor of  $C$  fd, the average current during a quarter cycle is

$$I_{av} = C \frac{E_{max}}{\frac{1}{4f}} \quad \text{or} \quad I_{av} = 4fCE_{max}$$



**Figure 4-6. Relationship Between Time and Frequency.**

In fact, it is true for the entire cycle because each quarter cycle has the same amount of voltage change per second. This equation should be simplified to eliminate having to work with both average and maximum terms in one operation. The relationship between average current and maximum current of a sine wave is then

$$I_{av} = \frac{2}{\pi} I_{max}.$$

You should recognize  $2/\pi$  as the familiar 0.637. Substituting this value for  $I_{av}$  in its equation,

$$\begin{aligned} \frac{2}{\pi} I_{max} &= 4fCE_{max}. \\ I_{max} &= 2\pi fCE_{max} \end{aligned}$$

Both sides of the above equation are in terms of maximum values and may be changed to effective values, giving

$$I = 2\pi fCE,$$

which indicates that the current flowing through a capacitor is proportional to the frequency of the applied sine wave and the capacitance. As in the case of resistive circuits, you find the opposition to the flow of current by dividing the voltage by the current. Then by equation  $I = 2\pi fCE$ .

$$\frac{E}{I} = \frac{1}{2\pi fC}.$$

where  $f$  (frequency) is the Hz and  $C$  is capacitance in farads. The angular velocity  $2\pi f$  is often written  $\omega$  (omega), making reactance equal to  $1/\omega C$ . This equation shows that capacitive reactance is inversely proportional to both capacitance and frequency. The manner in which the opposition to current by a capacitor ( $X_C$ ) changes with frequency is shown in the graph in figure 4-7. As  $f$  increases,  $X_C$  decreases along with the vertical axis. Ohm's Law is applied to capacitive reactance by substituting  $X_C$  for  $R$ ,

$$E = IX_C, \quad I = \frac{E}{X_C}, \quad X_C = \frac{E}{I}.$$

The capacitive reactance ( $X_C$ ) is the opposition offered to the flow of the current by a capacitor. Since the formula for capacitive reactance was found by working with a sinusoidal voltage applied, it follows that the formula

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi} \times \frac{1}{fC} = \frac{0.159}{fC}$$

is true only for sine wave voltages.

You can use the two methods discussed above for determining capacitive reactance by applying them to the circuit shown in figure 4-8. If you place AC voltmeters across  $R$  and  $C$  and insert an AC ammeter in the circuit you get the readings as indicated in the figure.

These values are actual meter readings. Now since you have the circuit current measured, you can calculate the capacitive reactance of  $C$  by

$$X_C = \frac{E}{I} = \frac{80V}{2 \text{ amps}} = 40\Omega.$$

Without knowing the circuit current, you can calculate capacitive reactance with

$$X_C = \frac{0.159}{fC} = \frac{0.159}{1590 \times 2.5 \times 10^{-6}} = \frac{10^{-4}}{2.5 \times 10^{-6}} = \frac{10^2}{2.5} = 40\Omega$$

Notice in figure 4-8 that more voltage is dropped across R and C than is applied from the generator. The introduction of the capacitor in the circuit has produced a strange circuit behavior. This is because the capacitor causes a phase shift between the voltage and current. Therefore you must use vectors to calculate circuit voltage and current. Vectors for series RC circuits will be discussed later.

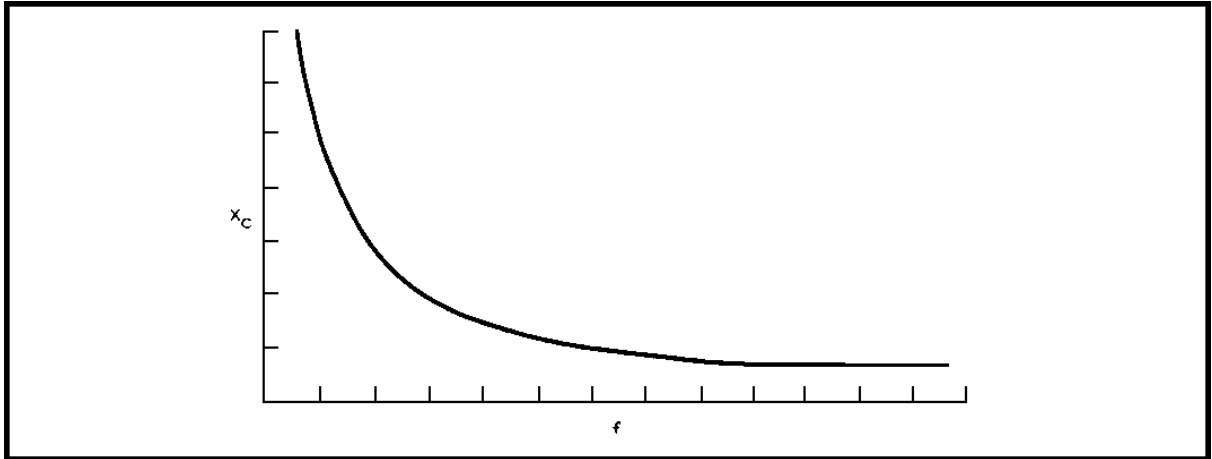


Figure 4-7. Relationship Between Capacitive Reactance ( $X_C$ ) and Frequency ( $f$ ).

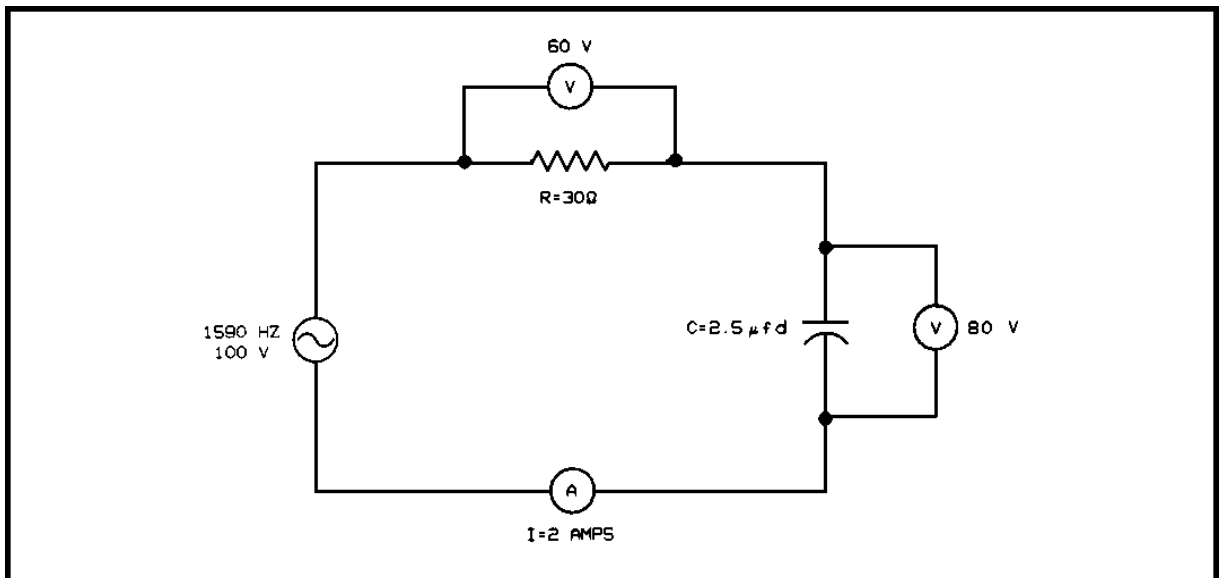


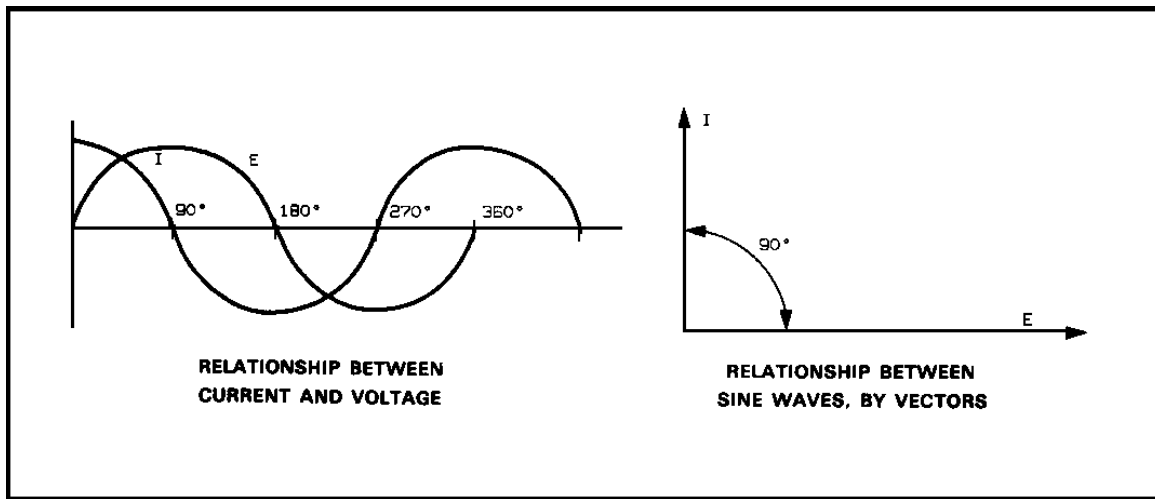
Figure 4-8. AC Voltmeters Across an RC Circuit.



### Phase Relationship in a Pure Capacitive Circuit

The current flowing through a capacitor at any instant depends on the rate at which the voltage across it is changing. Therefore, with a sine wave voltage applied, the current is maximum when the voltage is crossing the zero axis, because the voltage is then climbing or falling most rapidly. When the applied voltage is at its maximum, it is no longer climbing or falling. As it levels off, the current falls to zero. The relationship between current and voltage is shown in figure 4-9.

The voltage and current are seen to be one-quarter cycle, or  $90^\circ$  apart. Since the positive alternation of current reaches maximum before that of voltage, current leads the voltage. The phase or time relationship between these two sine waves is also shown by vectors in figure 4-9. The general statement can be made that when a sine wave of voltage is applied to a pure capacitance circuit, the current leads the voltage by  $90^\circ$ .



**Figure 4-9.** *Phase Relationship Between Current and Voltage in a Pure Capacitive Circuit.*

### Types of Capacitors

The major types of capacitors used in electronics and other electrical devices are, mica, ceramic, impregnated paper, air, and electrolytic.

**Mica.** The mica capacitor is thin metal plates separated by sheets of mica. The smaller sizes may be only two plates; larger sizes are numbers of plates and mica sheets. Alternate plates are connected together to provide a large plate area. The general arrangement is shown in figure 4-10. For electrodes, some mica capacitors use thin layers of silver deposited directly on the surface of the mica. A mica capacitor with molded bakelite case and wire terminals is shown in figure 4-11. Mica capacitors are made in small capacitances up to about  $0.5 \mu\text{fd}$ . They can be made for very high voltages, to a high degree of accuracy, and very stable under temperature changes. They are highly suitable for use in high frequency circuits.

**Ceramic.** Ceramic capacitors have most of the desirable qualities of mica capacitors and, in addition, may be made with a positive or negative temperature coefficient of capacitance. That is, the capacitance may be made to increase or decrease with changes in temperature. Such capacitors are very useful in compensating for temperature changes in nearby circuit components. The ceramic capacitor is made by depositing silver on the surfaces of a ceramic tube (figure 4-12).

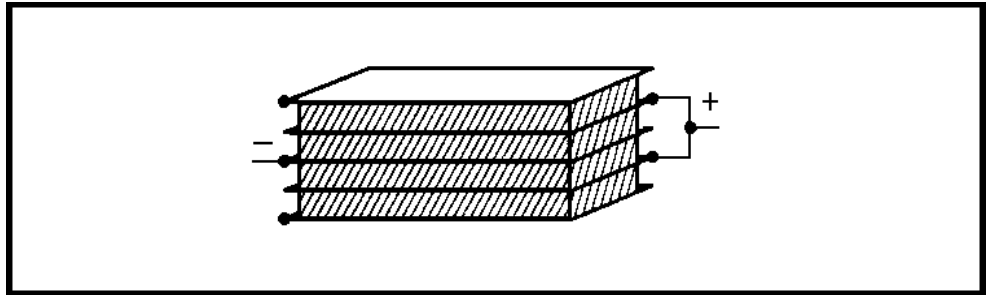


Figure 4-10. Mica Capacitor Showing Mica and Metal Sheets.

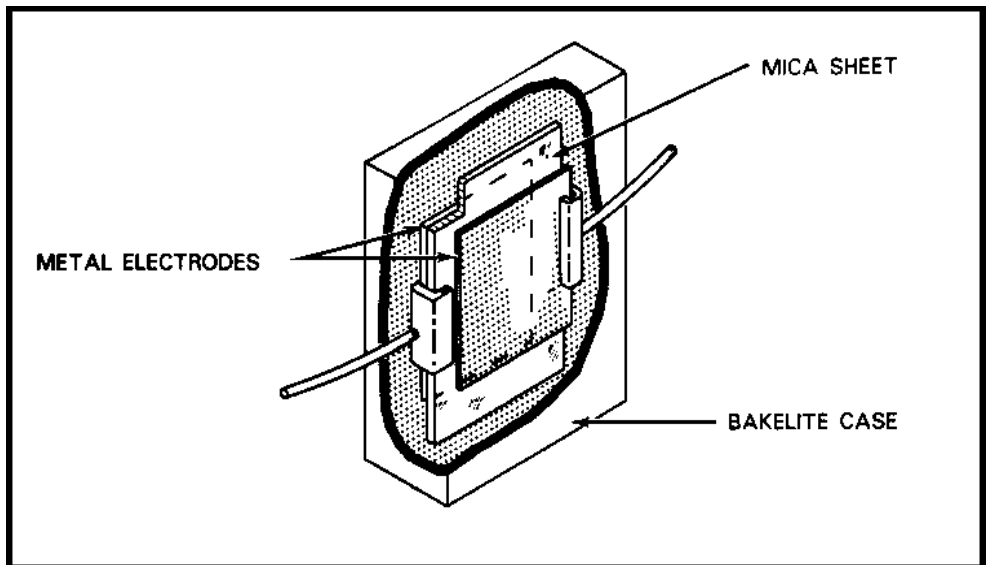


Figure 4-11. Molded Mica Capacitor (Enlarged).

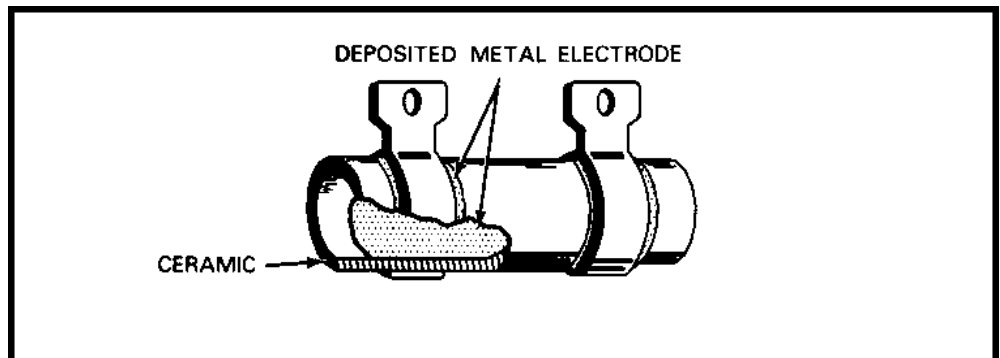


Figure 4-12. Ceramic Capacitor.

Ceramics are used in transmitters because they meet the exacting requirements of rigid frequency control. They are the only kind of capacitor used for holding oscillator frequencies to the close limits obtained by crystal control. (Such capacitors are usually negative temperature coefficients.) Hermetically sealed ceramic capacitors have been developed for the precision circuitry of electronic instruments. These are hermetically sealed to withstand changes in atmospheric moisture. Also, they will withstand the vibration and shock normally encountered in military electronic equipment. These capacitors permit capacitance tolerances within plus or minus 1 percent and temperature coefficient tolerances within plus or minus 10 parts per million, per degree Celsius.

**Impregnated Paper.** Probably the most used capacitor has impregnated paper as the dielectric. The paper is saturated with any number of resins, waxes, oils, or synthetic compounds. For conservation of space and for ease of manufacture, these capacitors are made by winding up two strips of metal foil separated by sheets of paper. To reduce the danger of breakdown because of flaws in the paper, several laminated sheets of paper are used, instead of a single, thicker sheet. The units must be carefully sealed to prevent the paper from absorbing moisture. When made for high voltages, the case of the capacitor is often filled with a high-grade mineral oil. Typical paper capacitors are shown in figure 4-13. The paper capacitor has a greater ratio of capacitance to weight than the mica or ceramic type. It is suitable for power and audio frequencies but has excessive losses at higher radio frequencies.

**Air.** Air is used as the dielectric to make the capacitor variable. Figure 4-14 shows the construction of two kinds. The rotor plates are all fixed to a common movable shaft. The stationary (stator) plates are fastened to fixed terminals. There may be one or two sets of stator plates. Because of the large spacing between plates and the low dielectric constant, air capacitors are rather bulky and are rarely made larger than 500 pfd. They have low losses and are highly suitable for radio frequencies. An advantage is that the dielectric is self-healing. That means, that after an excessive voltage has been applied, the capacitor can be used again.

**Electrolytic.** The electrolytic capacitor is important because it provides a very large capacitance in a small space. It is two metal plates separated by an electrolyte. The electrolyte is the negative electrode. The dielectric in an electrolytic capacitor is a very thin oxide film formed on the surface of the positive capacitor plate. This allows a high capacitance in a small space because of the thin dielectric. Electrolytic capacitors can be used in AC or DC circuits, but the construction for each differs. DC capacitors are commonly marked to indicate polarity, which you must carefully observe. A reversal of applied voltage would destroy the dielectric film. AC electrolytic capacitors are usually used only for intermittent operation. A good example is the electric refrigerator's starting motor. Capacitance ratings of electrolytic capacitors are quite high; the smallest readily available size being 8  $\mu$ fd, while larger values run into thousands of microfarads. Voltage ratings are limited to about 450 working V. They are not suitable for use in critical circuits, since the capacitance varies greatly with temperature and age. In addition, they have considerable current leakage. Electrolytic capacitors are self-healing if the breakdown current is not too large. A dry electrolytic capacitor is shown in figure 4-15. Cheaper units are also supplied in cardboard containers with wire leads.

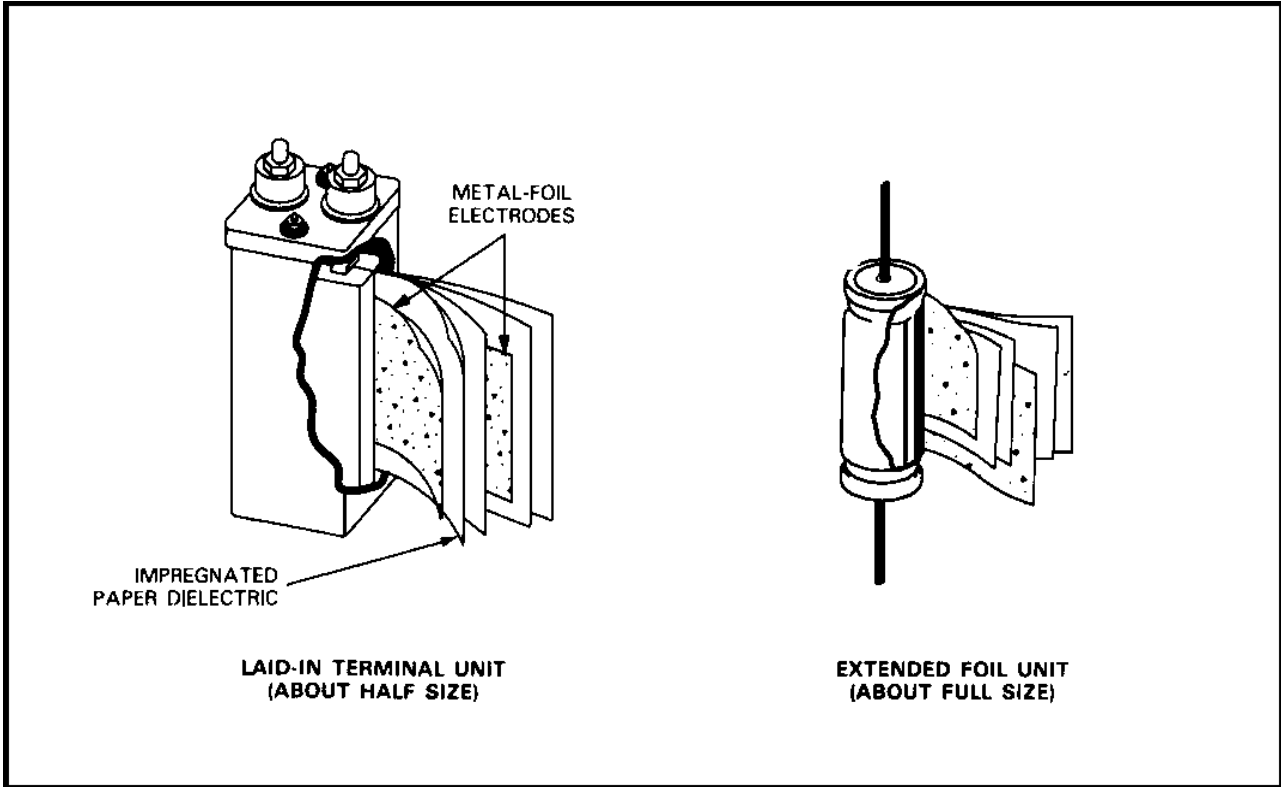


Figure 4-13. Impregnated Paper Capacitors.

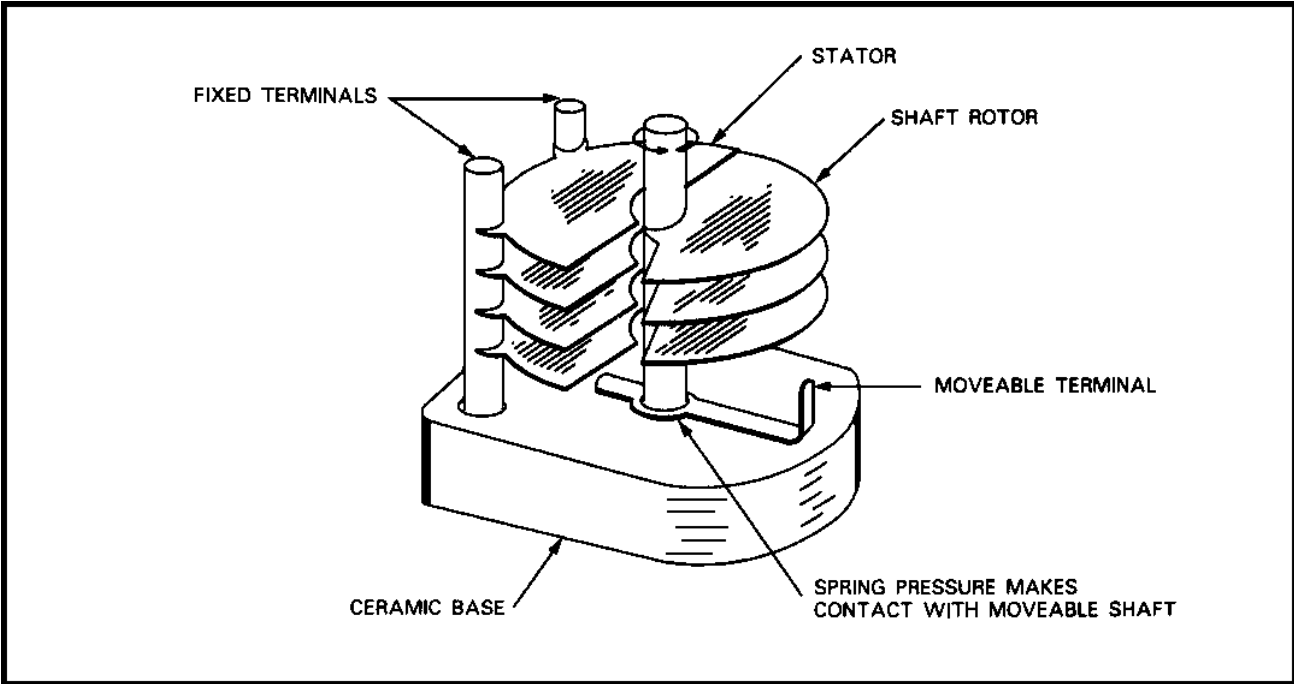
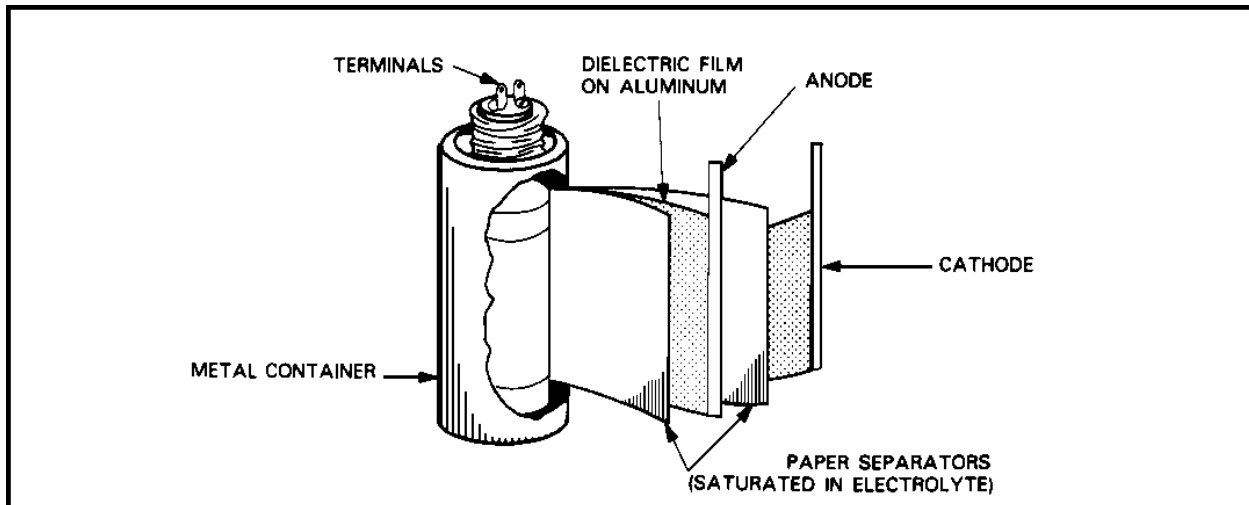


Figure 4-14. Variable Air Capacitor.



**Figure 4-15. Electrolytic Capacitor.**

### INDUCTANCE

The magnetic field windings of a large industrial DC or AC generator are energized with a few amperes of DC at about 220 V. Currents and voltages of this order can usually be interrupted by a switch no larger than the one that turns on the light in your room. Yet, to open the generator field circuit suddenly, even by means of a large and substantially built switch, would probably result in a violent arc of several thousand volts that would melt the switch and destroy the insulation. These results are highly out of proportion to the small amount of energy in the circuit. Where did enough come from to produce such destruction? The answer lies in the ability of a coil to store electrical energy. This property is called inductance.

The following experiments will help you understand inductance. A wire is wound on a cardboard tube as shown in figure 4-16. The ends of the wire are connected to a galvanometer, which is a sensitive measuring device. When a permanent bar magnet is suddenly moved into the coil as shown in figure 4-17, the galvanometer pointer will swing to the right, which indicates that a voltage has been generated and a current forced around the circuit. When the magnet is at rest, there is no indication on the meter. When the bar magnet is suddenly pulled out, the meter needle swings to the left. This time, the direction of the current has been reversed.

The above experiment shows that:

- Relative motion between a coil and a magnetic field produces voltage.
- The polarity of the voltage reverses when the relative motion is reversed.

The direction of current in the coil and the direction of the magnetic field that results from that current, demonstrates Lenz's Law:

- The countervoltage induced in a coil opposes the force which induced it.
- The change of current in a coil sets up a countercurrent opposing this change.

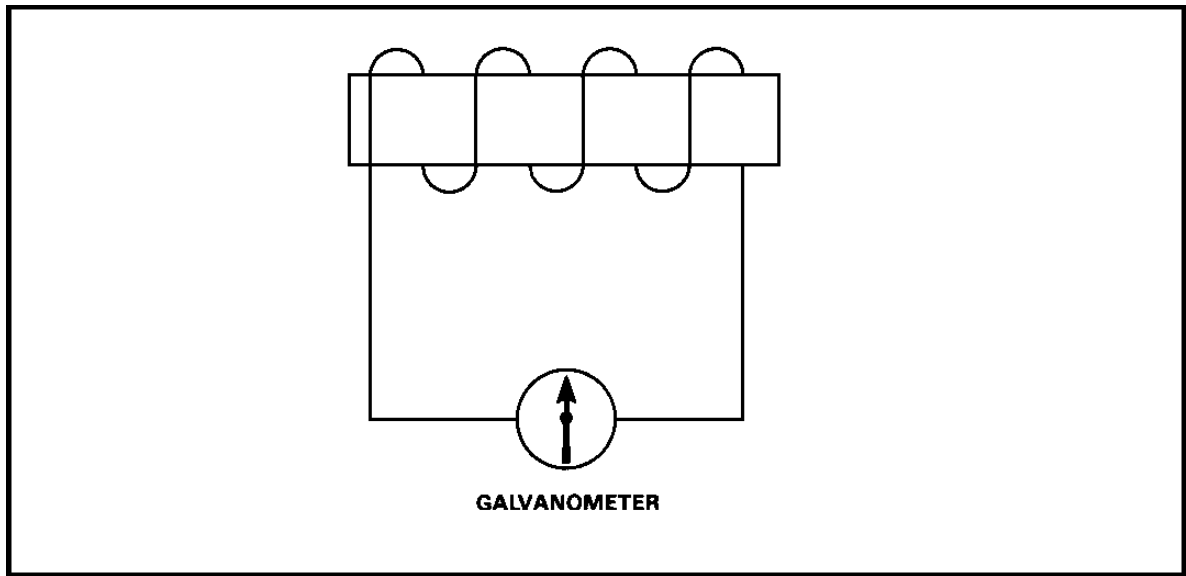


Figure 4-16. Coil of Wire With Current Measuring Device.

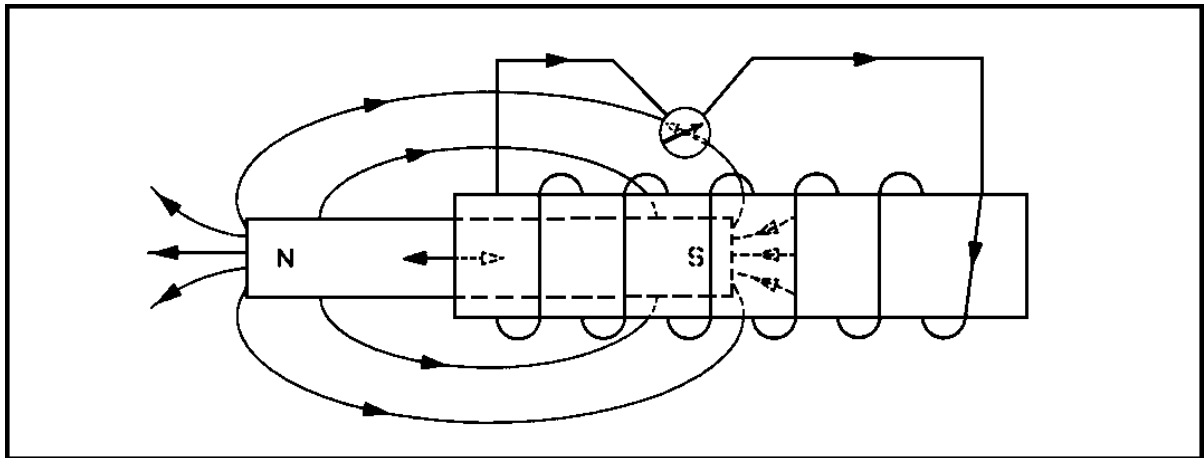


Figure 4-17. Inducing Current in a Coil.

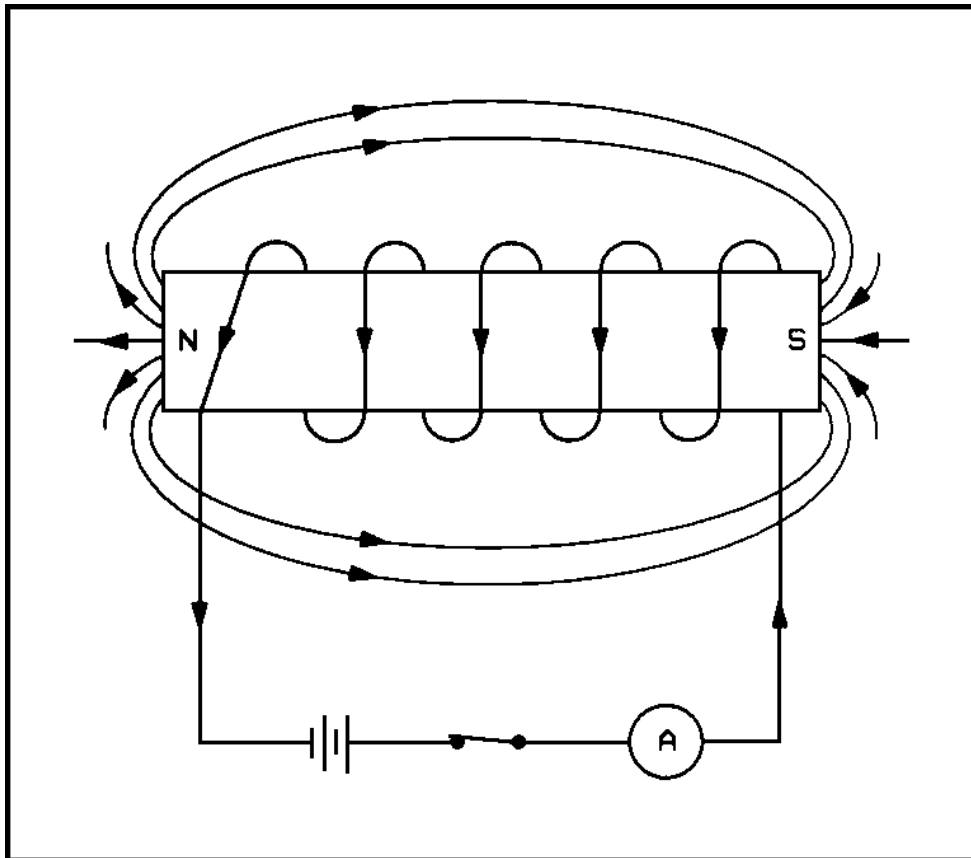
From the observations in the previous experiment, you note that the swing of the galvanometer pointer is proportional to the rate at which the magnet is moved. In other words, the voltage induced in the coil is proportional to the rate of change of the flux.

Assuming that the volume of the coil does not change, if the number of turns were doubled the current would increase four times. The induced voltage, which is proportional to the current, must then be proportional to the square of the number of turns, and directly proportional to the rate of change of the field.

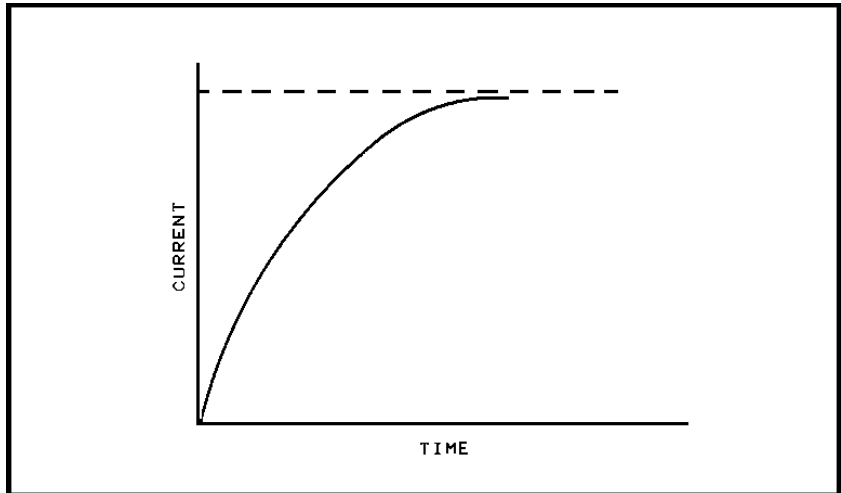
Changing the current through a coil has an important effect upon its operation. Figures 4-18 and 4-19 explain this effect.

When the switch is closed (fig 4-18), current flows and a magnetic field is set up as shown. Since this field is identical to one produced by a bar magnet, it would be reasonable to expect that when this magnetic field is set up, an opposing reaction (Lenz's Law) would occur similar to that observed when the bar magnet was inserted into the coil. That this reaction does occur may be observed when you use an ammeter (with very little inertia), which will respond almost instantaneously to the current flow. Such a meter shows that the current does not jump suddenly to a maximum, but rises at a very definite rate, gradually approaching a final value. Figure 4-19 shows how the current in the coil rises with time. This slow rise in current is caused by self-induction, or countervoltage.

When the switch is closed, the resulting change induces a countervoltage in the coil that opposes the change in current flow. The countervoltage induced is proportional to the rate of change of the current. The greatest rate of current change occurs the instant the switch is closed, since at that instant, the current is changing from zero to some definite value. Also at that instant, the counter electromagnetic force (EMF) is the greatest. As the current levels off to its final value, the field ceases to change and the voltage falls to zero. The current is now limited only by the resistance in the circuit (in spite of the fact that the field is quite strong) because the field is no longer changing.



**Figure 4-18. Magnetic Field of a Coil Carrying a Current.**



**Figure 4-19. Buildup of a Current in a Coil.**

It took energy to build up this magnetic field. Most of this energy is stored in the field around the coil and is returned to the circuit when the source voltage is removed. A small amount of the original energy is lost as heat because of the resistance in the coil. When the source of energy is removed, this stored energy acts to keep current flowing in the same direction. If a battery as a source of energy is replaced with a low resistance, the ammeter will not fall immediately to zero but will show that the current takes time to decline, falling quickly at first and finally leveling off to zero. The direction of current flow does not change. This is because a reversed voltage is generated across the coil during the collapse of the field.

The effect of a coil (its inductance) is much the same as mechanical inertia. Just as it takes time and energy to start a flywheel, it takes time and energy to start a current in an inductive circuit. And, just as the flywheel is capable of giving up energy as it coasts to a stop, inductance returns its stored energy as the current decreases. When a flywheel is stopped suddenly, its stored energy produces destructive forces. Similarly, if an inductive circuit is opened suddenly, the rapid collapse of the field produces a large voltage across the coil. This keeps the current flowing, and the stored energy is almost instantaneously converted to heat at the opening switch contacts. If the inductance and the current are large, the contacts may be badly burned or the insulation of the coil damaged.

#### **Unit of Induction**

Since the effects of inductance are due to the voltage induced when the current through it is changed, the unit of inductance can be defined as follows:

A coil of wire has an inductance of one henry if a current change of 1 amp per second will cause a pressure of one V to be set up in the coil.

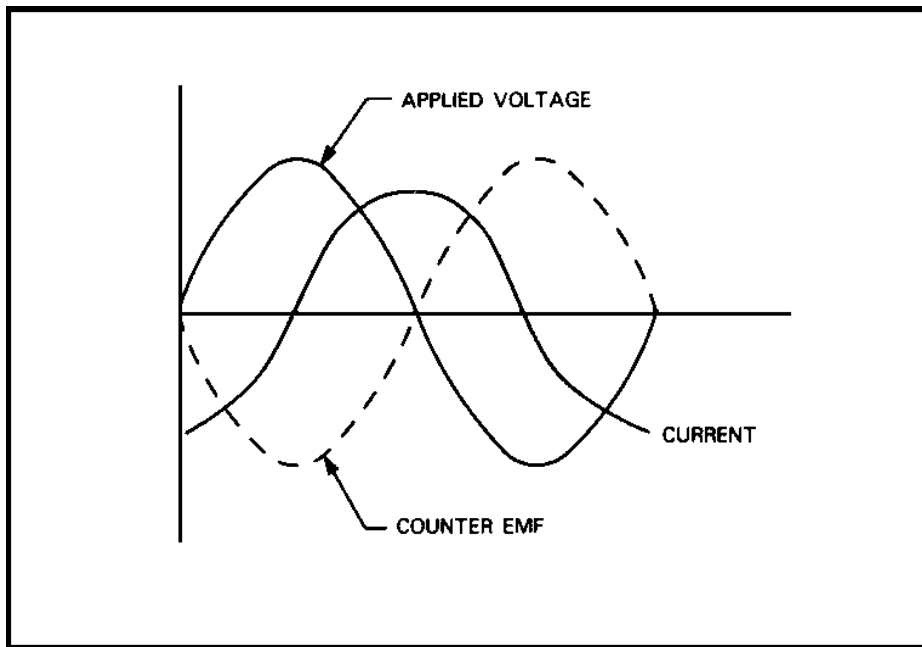
Inductances are rated in henrys, millihenrys, and microhenrys. A millihenry (mh) equals  $10^{-3}$  h, and a microhenry ( $\mu$ h) equals  $10^{-6}$  h.



### Phase Relationship in a Pure Inductance

Unlike capacitors, which for most purposes contain negligible series resistance, a coil (inductor) contains a considerable amount of resistance. This is due to the resistance of the wire and to eddy currents and hysteresis losses in the core. (Hysteresis is the lagging of the magnetic flux lines behind the magnetizing force.) A commercially available 30-h coil may have from 100 to 500  $\Omega$  of DC resistance. Pure inductance is not considered physically possible. However, when the inductive reactance (opposition of the coil to a change in current) is large compared to the resistance, it is safe to treat a coil as though it were a pure inductance.

With a sine wave of current flowing in an inductance, the countervoltage is greatest, not when the current is greatest, but when the current is changing to the greatest rate. The counter EMF reaches a maximum negative value when the current is increasing most rapidly in a positive direction (figure 4-20).



**Figure 4-20. Applied Voltage, Counter EMF, and Current Across an Inductor.**

The counter EMF is at all times opposed to the applied voltage that is making the current flow (Lenz's Law). That is, the back voltage is 180° out of phase with the applied voltage. You can see from the position of the waves along the axis, that the current, in a pure inductance, lags the applied voltage by 90°. The counter EMF is exactly opposite to the applied voltage.

### Inductive Reactance

Because inductance tries to prevent a change in the current flowing through it, a coil will show considerable opposition to the flow of an alternating current. The exact amount of opposition to an alternating current sine wave may

be derived from the fundamental relationship between voltage, inductance, and rate of change of the current. That is,

$$E_L = \frac{-L \Delta i \text{ (Change in current)}}{\Delta t \text{ (Change in time)}},$$

where the quantities are in basic units; volts, henrys, amps, and seconds.

The time for one cycle of an AC wave is  $1/f$  (figure 4-21). The time required for a quarter cycle (which is chosen because the wave from zero to maximum can be examined) is  $1/4f$ . If a sine wave of current with a peak value of  $I_{\max}$  and a frequency of  $f$  is sent through an inductance of  $L$  h, the average voltage across the coil during a quarter cycle is:

$$E_{\text{av}} = L \frac{I_{\max}}{\frac{1}{4f}}.$$

To simplify the equation into a more usable form, convert

$$E_{\text{av}} = \frac{2}{\pi} E_{\max}.$$

Substituting this value for  $E_{\text{av}}$  in the above equation,

$$E_{\max} = 4fL I_{\max} \times \frac{\pi}{2}$$

and

$$E_{\max} = 2\pi fL I_{\max}.$$

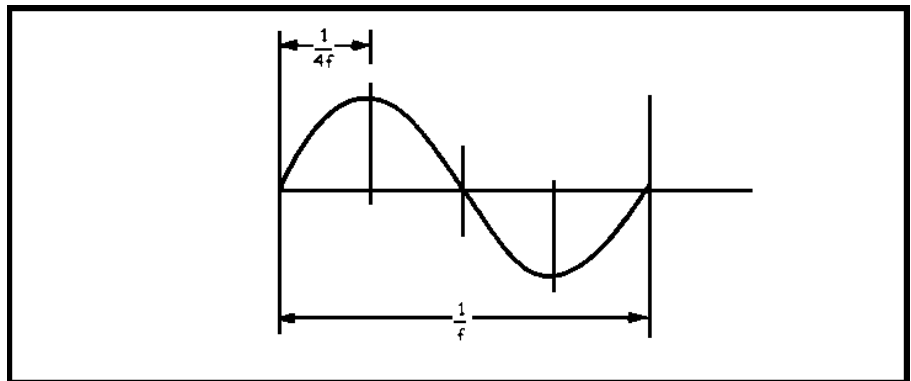
Since both sides of the equation are in terms of maximum values, the subscripts may be dropped and written in terms of effective values. Thus,

$$E = 2\pi fL I.$$

shows that the voltage drop across an inductance is directly proportional to the frequency, inductance, and current.

The opposition to the flow of current is, as with an electrical device, equal to  $E/I$ . Rearranging the above equation, we get

$$\frac{E}{I} = 2\pi fL.$$



*Figure 4-21. One AC Wave Cycle.*

The opposition to the flow of an alternating current by an inductance is called inductive reactance, denoted by  $X_L$ , and is measured in ohms. The equation then becomes

$$X_L = 2\pi fL,$$

or in shorter form, letting omega equal  $2\pi f$ ,

$$X_L = \omega L.$$

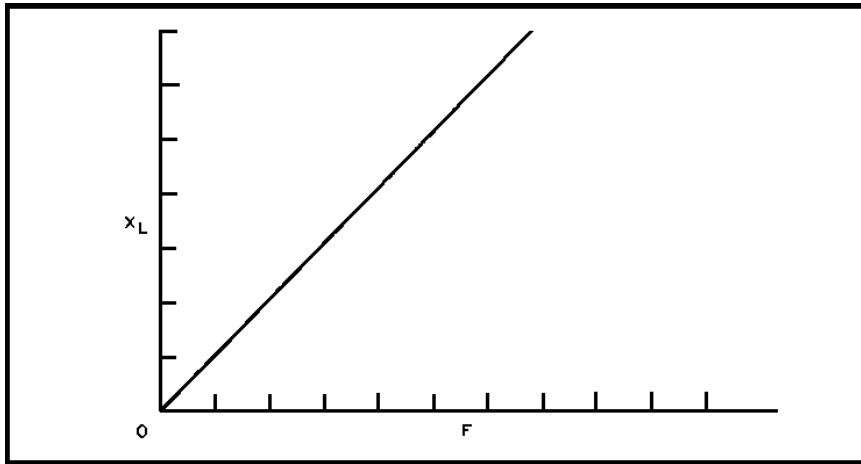
Inductive reactance is directly proportional to inductance and frequency. This is extremely important. It makes it possible to separate DC from AC in the same circuit or low frequencies from high frequencies. When used in this connection, an inductance is often called a choke. How  $X_L$  varies with frequency is illustrated by the graph in figure 4-22.

Ohm's law is applied to inductive reactance as shown.

$$E = IX_L$$

$$I = \frac{E}{X_L}$$

$$X_L = \frac{E}{I}$$



**Figure 4-22. Variation of  $X_L$  With Frequency.**

#### **Series and Parallel Connection of Inductors**

Rules for connection of inductors (series or parallel) are based on the assumption that there is no coupling between the individual inductors; that is, lines of magnetic force from one do not induce an appreciable voltage in another. Stray coupling is generally negligible between adjacent laminated iron units because the flux is well confined to the core. Air core coils are isolated by spacing them sufficiently or by placing them at right angles to each other.

Inductances connected in series are added similarly to resistances.

$$L_t = L_1 + L_2 + L_3 + \dots + L_n$$

Inductances connected in parallel are also combined in the manner of resistances.

$$\frac{1}{L_t} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \dots + \frac{1}{L_n}$$

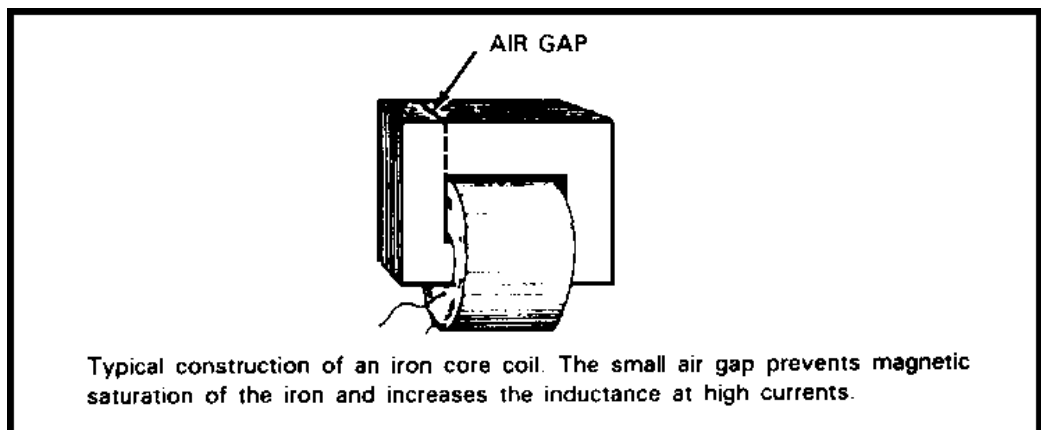
The voltage drop across each series inductor, with a changing voltage or alternating current applied, is directly proportional to L or to XL. With parallel connections, the voltage across each coil is the same, but the current is distributed in inverse proportion of L or XL.

### **Factors Determining Size of Inductance**

In general, a coil arranged to produce a large amount of magnetic flux will also have a large inductance. The amount of inductance depends on the number of turns, the length, cross-sectional area, and permeability of the magnetic path.

Air core inductance is a term applied to coils having cores of nonmagnetic materials, such as air, wood, or ceramics. The inductance, which seldom exceeds a few hundred millihenrys, is wound in a number of forms. It may consist of a single layer winding on an insulating tube; it may have a large number of layers of wire, or it may be wound with a heavy self-supporting conductor with little or no solid supporting insulation. It is a good idea to reduce the distributed capacitance which results from turns being close to each other. A number of multilayer winding arrangements have been developed that reduce the shunt capacitance considerably.

Iron core inductors are used when large values of inductance are needed. They are made as large as several hundred henrys. Their larger inductance results from a magnetic path of low reluctance, which is a laminated iron core in the form of a closed magnetic circuit. The addition of such an iron core may easily increase the inductance of the coil a thousand times. Iron core inductors are rated for their current-carrying capacity, as well as for their inductance. Typical construction is shown in figure 4-23. Since an iron core coil is generally used in AC or pulsating DC circuits, it must be laminated. With a continually changing flux, a solid core would have voltages induced in it that would cause heavy circulating currents called eddy currents. This would result in heating of the core and losing power. Laminations limit eddy currents to a small value and reduce the power loss.



*Figure 4-23. Iron Core Inductor.*

The inductance of an iron core coil is not constant. This is because the permeability of the core (degree to which the core is magnetized) varies as the magnetizing force changes, due to the current flowing through the core. In addition, there is magnetic saturation of the core at high flux densities. A small air gap placed in the core prevents magnetic saturation of the iron and makes it possible to use the coil over a wide range of current flow.

The inductance of an iron core coil, like that of the air core, is dependent on the diameter and the number of turns. In addition, the permeability of the core is a factor. Since the permeability is not constant, the current flow and magnetic current in the circuit must be considered in figuring the inductance.

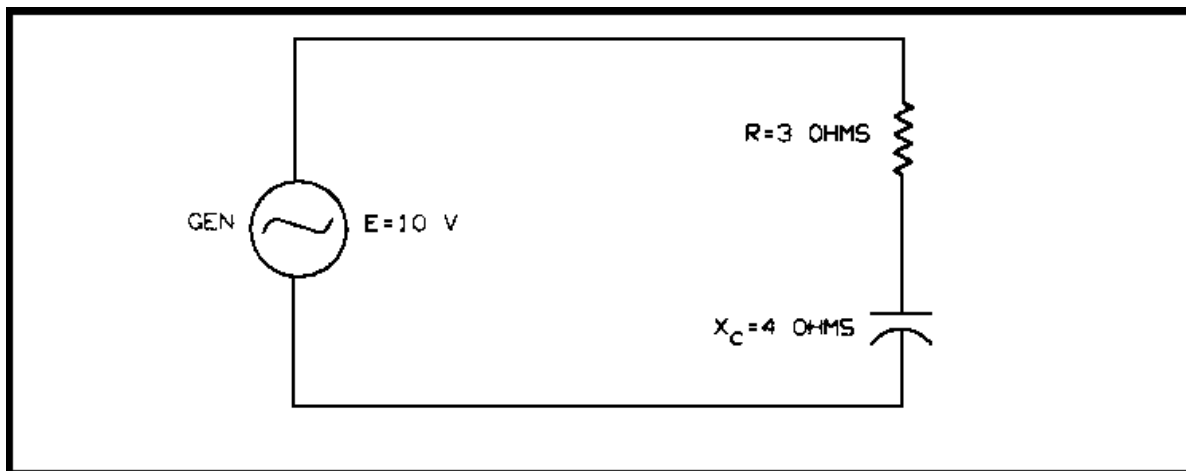
The laminated core is not suitable for radio frequencies because of eddy current losses. While eddy currents can be reduced by the use of very thin laminations, it is not practical to produce iron sheets thinner than 0.001 in. Laminated iron core coils are useful only at frequencies lower than the radio frequency range. However, by subdividing the iron into a very fine dust, it is possible to produce an iron core suitable for radio frequencies as high as 30 MHz. The iron is suspended in a plastic insulating material, forming a plug that is placed inside the coil. The inductance is readily made variable by adjusting the position of the core.

#### **SERIES RESISTIVE AND CAPACITIVE CIRCUITS**

In a circuit containing only resistance, the voltage across the resistance and the current through the resistance are in phase; in a circuit containing only reactance, the voltage and current are 90° out of phase. However, some circuits contain both resistance and capacitive reactance (RC).

A series RC circuit is shown in figure 4-24, where an AC voltage of 10 V is impressed across a combination of 3 Ω of resistance in series with 4 Ω of capacitive reactance. This circuit can be used to illustrate most explanations of a simple series RC circuit.

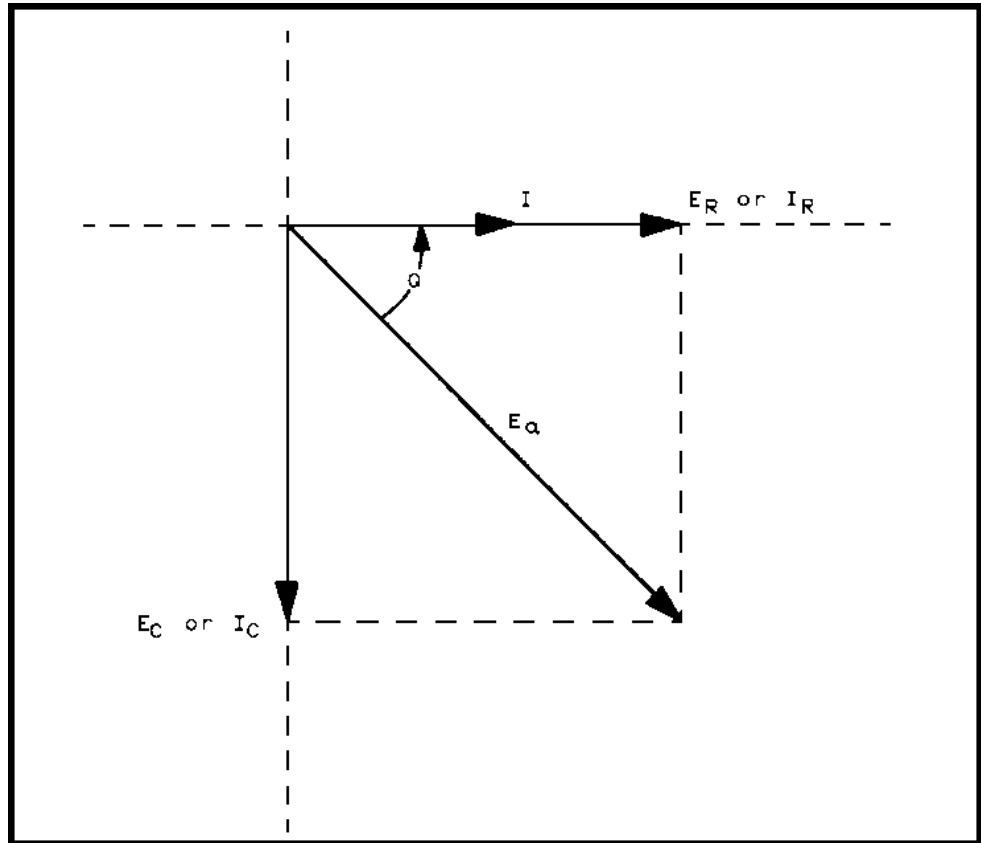
As in DC circuits, the sum of the voltage drops around the circuit comprising the load must equal the applied voltage. However, in considering resistance and reactance, voltages can no longer be added or subtracted arithmetically.



**Figure 4-24. Series RC Circuit.**

This is because the voltage drop across the resistance is in phase with the current, but the voltage drop across the capacitive reactance is 90 degrees behind the current.

The combination of capacitance and resistance in the circuit causes the current to lead the applied voltage by some angle less than 90°. Therefore, the current vector is drawn in the first quadrant as shown in figure 4-25. The voltage across the resistor ( $IR$ ), since it is in phase with the current, is drawn along the same line as the current vector. Because the voltage across the capacitor ( $IX_C$ ) lags the current by 90°, it is drawn downward at an angle of 90° with the current and resistance voltage vectors.



**Figure 4-25. Current and Voltage Vectors for RC Circuits.**

The applied voltage is the vector sum of these two voltages. This vector sum is the diagonal of the rectangle whose sides are  $E_R$  and  $E_C$ . Because the diagonal divides the rectangle into two congruent right triangles, then:

$$E_a = \sqrt{E_R^2 + E_C^2}$$

$$E_a = \sqrt{(IR)^2 + (IX_C)^2}$$

$$E_a = \sqrt{I^2R^2 + I^2X_C^2}$$

$$E_a = I\sqrt{R^2 + X_C^2}$$

$$\frac{E_a}{I} = \sqrt{R^2 + X_C^2}$$

The quantity  $\sqrt{R^2 + X_C^2}$  is the total opposition, the result of having the two oppositions to current flow (R and  $X_C$ ) in the same circuit. This total opposition to AC current flow is called the impedance of the circuit and is shown by Z. Since Z is an opposition to current flow, it is measured in ohms.

The impedance in any circuit can always be found if the total voltage and the total current are known. Ohm's law for AC circuits states that the impedance of the circuit is equal to the total voltage divided by the total current. Expressed as a formula, this would be

$$Z = \frac{E_a}{I}.$$

By transposing, this formula would be

$$I = \frac{E_a}{Z}.$$

or it could be

$$E_a = IZ.$$

It is also possible to find the impedance if the reactance and resistance of the circuit are known.

$$Z = \frac{E_a}{I}$$

and

$$\frac{E_a}{I} = \sqrt{R^2 + X_C^2}$$

Then it must follow that,

$$Z = \sqrt{R^2 + X_C^2}$$

or

$$Z = \sqrt{R^2 + X_L^2}.$$

### **Power Dissipation in RC Circuits**

Now what about power in the circuit? When current flows through a resistor, power is dissipated in the form of heat. For example, the power used by the resistor can be found by using the formula  $P=I^2R$ .

The capacitor, however, uses no power because a capacitor stores energy rather than uses it. When an alternating force is impressed across a capacitor, power is taken from the source and energy is stored in the capacitor as the voltage increases from zero to a maximum positive value. As the voltage decreases from a maximum positive value to zero, the capacitor discharges and returns this energy to the source. Likewise, during the negative half-cycle, while the voltage varies from zero to a maximum negative value, the capacitor in charging takes energy from the source. This energy is returned to the source as the cycle is completed. During each cycle, energy is transferred back



and forth but none is used. Actually, not quite as much is returned to the source by the capacitor on the discharge as is taken on the charge. The small amount of energy that is not returned to the source is dissipated by the DC resistance of the connection. The DC loss, in comparison with the amount of energy stored, is so small that for all practical purposes the capacitor uses no power.

If a voltmeter is placed in parallel with the capacitor, and a current meter in series with the capacitor, it seems that the capacitor is using power. This power is called reactive power and is given by the relation

$$P = I^2 X_C.$$

Power is equal to the voltage times the current in a pure resistive circuit. However, in a reactive circuit, this relationship is a false indication and is called apparent power.

$$P_a = EI$$

The actual power taken by an AC circuit is termed the true power. The formula for it is

$$P_{\text{true}} = I^2 R,$$

or

$$P = E_R I,$$

where  $E_R$  is the potential difference across the resistance of the circuit and  $R$  is the resistance in the circuit.

Since the apparent power is a false indication, the power factor (P.f.) is the ratio of the true power to the apparent power. That is,

$$\text{p. f.} = \frac{P_{\text{true}}}{P_a}.$$

$$\text{P. f.} = \frac{I^2 R}{EI} = \frac{IR}{E}.$$

Then

$$E = IZ,$$

$$\text{p. f.} = \frac{IR}{IZ}, \quad \text{and}$$

$$\text{p. f.} = \frac{R}{Z}.$$

Hence, you can get the power factor of a series circuit by dividing the resistance of a circuit by its impedance. Reviewing the impedance vector diagram and the current and voltage diagram, it is true that the trigonometric relationship of  $R/Z$  is  $\cos \theta$ , where  $\theta$  is the angle by which the current leads or lags the applied voltage.

Therefore,

$$\text{p. f.} = \cos \theta.$$

In many circuits, it is desirable to find the true power when the applied voltage, the current, and the phase angle are known. The equation for the true power can be derived from the definition of power factor,

$$P = P_a \text{p.f.}$$

Then

$$P = EI \text{ p.f.},$$

and

$$P = EI \cos \theta.$$

To find the true power in an AC circuit, any of the following formulas may be used.

$$P = I^2R$$

$$P = E_R I$$

$$P = EI \cos \theta$$

### **Effect of Frequency Change in RC Circuits**

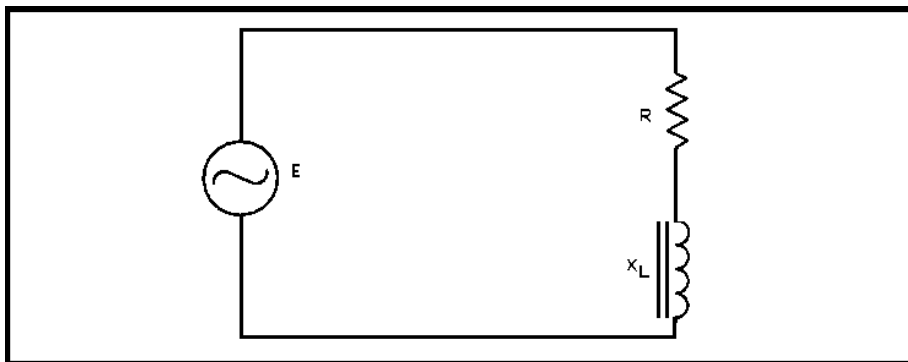
In many AC circuits, the value of the capacitor and the frequency of the applied voltage are given; from this, the capacitive reactance may be found using the formula:

$$X_c = \frac{1}{2\pi fC}.$$

From the above formula, the capacitive reactance is inversely proportional to the frequency. Doubling the frequency of the applied voltage in this problem will reduce the capacitive reactance by one half.

### **SERIES RESISTIVE AND INDUCTIVE CIRCUITS**

In the discussion of inductance, you learned that the current through an inductance lags the voltage across it by  $90^\circ$ . Also, you learned that the current through a resistance is in phase with the voltage across the resistance. What happens when resistance and inductance are in series in an AC circuit? Such a circuit is shown in figure 4-26, where an alternating EMF is impressed across a combination of resistance in series with inductive reactance.



**Figure 4-26. Series RL Circuit.**

Although all coils have some resistance in their winding, this resistance is small compared with the reactance of the coil. For this reason, the circuit can be considered just two components, a pure inductance and a pure resistance.

In a series RL circuit, the following phase relationships are always true:

- The current lags the applied voltage.
- The voltage across the resistor lags the applied voltage.
- The voltage across the coil leads the applied voltage.
- The voltage across the coil leads the voltage across the resistor by 90°.

The angles, by which the current and voltage-drops lag or lead the applied voltage, are determined by the size of the coil, the value of the resistor, and the frequency of the applied voltage. Changing any of these three factors will not change any of the above statements, but will change the size of the phase angles and the magnitude of the current and voltage-drops.

### **Power Dissipation in RL Circuits**

The coil resembles the capacitor as far as power dissipation is concerned. When an alternating current is flowing through an inductor, energy is being taken from the circuit during the quarter cycle that the current is increasing. This energy is stored in the magnetic field. As the current decreases, the magnetic field collapses. The collapsing magnetic field induces a counter EMF, thereby returning the energy to the circuit. This process of storing the energy in the field and then returning it to the circuit happens every half cycle. The overall result is that the energy is transferred back and forth, but none of it is used. Actually, the wire in the coil has some resistance; so, as the current flows back and forth through the coil, some power is dissipated. This power is so small, compared with the energy being stored, for practical purposes, an inductor uses no power.

This apparent use of power is termed "reactive power," just as in the case of capacitors. However, the reactive power of the capacitor is stored in the dielectric field while that of the coil is stored in the magnetic field. The formula for apparent power,

$$P_a = EI,$$

is applied to any AC circuit, whether it contains capacitive reactance or inductive reactance, or a combination of resistance and reactance. The true power used by an RL circuit is found by the same formula as the true power in the RC circuit.

A summary of phase relationships in AC circuits is in the following paragraphs. A knowledge of these relationships will greatly aid you in understanding the subject of alternating current.

In a series RC circuit--

- The current leads the applied voltage.
- The voltage across the resistor leads the applied voltage.
- The voltage across the capacitor lags the applied voltage.
- The voltage across the capacitor lags the voltage across the resistor by 90°.

In a series RL circuit--

- The current lags the applied voltage.
- The voltage across the resistor lags the applied voltage.
- The voltage across the coil leads the applied voltage.
- The voltage across the coil leads the voltage across the resistor by  $90^\circ$ .

#### **Effect of Frequency Change in RL Circuits**

Increasing the frequency in a series RL circuit causes the reactance, the impedance, and the voltage across the coil to increase; at the same time, the current in the inductor decreases.

Doubling the inductance would have the same effect as doubling the frequency. Reducing the resistance one half would also have the same effect on phase relationship, but the current would be greater.